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Exact Solutions of the Schrodinger Equation for the Inverse Quadratic Yukawa Potential using Nikiforov-Uvarov Method

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ABSTRACT

The bound state solutions of the Schrodinger equation have been obtained analytically for the inverse quadratic Yukawa potential. The Nikiforov-Uvarov method was employed to obtain the energy eigenvalues and the corresponding eigen functions expressed in terms of the Jacobi polynomials.We discussed the variation of the energy spectrum as a function of the quantum number n and the atomic number Z

Keywords

Schrodinger equation, Inverse quadratic, Yukawa potential, Nikiforov-Uvarov, Eigen function

1. INTRODUCTION

It is interesting way any l-state solutions of the Schrodinger equation can be obtained for several diatomic molecules within a given potential [1-5].The Schrodinger equation reveals that the eigen function of the system can furnish us with information regarding the behavior of such a physical system. Therefore, if an exact solution of the Schrodinger equation is obtained for such a system, an energy eigen values or eigen function can describe such a system completely. However, the exact analytical solutions of the Schrodinger equation are only possible with the angular momentum $l = 0$ for some potentials [1,6-8]. When $l \neq 0$,the Schrodinger equation can only be solved approximately using different suitable approximation schemes [6-10].Many authors have investigated the bound state solutions of the Schrodinger equation approximately for certain potentials. For example, Arda and Sever[11] have obtained bound state solutions of the Schrodinger equation for generalized Morse potential with position dependent mass. Also, Ikot and Akpabio[6] investigated the approximate analytical solution of the Schrodinger with the Hulthen potential for arbitrary l −state. In their work a new approximation scheme was introduced for the centrifugal term. Koc and Haydargil [8] solved the Schrodinger equation with one and two dimensional double wall-well potentials in the framework of the $SL_2(R)$ lie algebra and obtained the eigen values and corresponding eigen function in terms of the orthogonal polynomials. The exact solutions of the D-dimensional Schrodinger equation with pseudo-coulomb potential plus ring-shaped potential has also been obtained by Ikhdair [12] to mention but a few literature.

In this paper,the Schrodinger equation has been solved for an inverse quadratic Yukawa potential V(r) of the form:

$$V(r) = \frac{Z}{r^2} e^{-2br}, \qquad (1)$$

Where b is screening parameter and Z is the atomic number. A form of the Yukawa potential has been earlier used by Taseli [13] in obtaining modified Laguerre basis for hydrogen-like systems. Also Kermode etal [14] have used different forms of the Yukawa potential to obtain the effective range functions. But not much has been done in solving the Schrodinger equation (SE) for the Inverse quadratic Yukawa potential (IQY). We explore the Nikiforov-Uvarov(NU) method in getting the energy eigen values and corresponding eigen functions for IQY with arbitrary l −state.

The paper is organized as follows: In section II, The NU method is reviewed. In section III, we solve the radial Schrodinger equation for IQY potential for an arbitrary angular momentum. Section IV is devoted for discussion. Finally, we give a brief conclusion in section V

2.0 REVIEW OF NIKIFOROV-UVAROV METHOD

The NU method [15] was proposed and applied to reduce the second order differential equation to the hypergeometric – type equation[11] by an appropriate co-ordinate transformation S = S(r) as [16].

$$\varphi''(s)+\frac{\bar{z}(s)}{\sigma(s)}\varphi'(s)+\frac{\bar{\sigma}(s)}{\sigma^2(s)}\varphi(s)=0 \qquad (2)$$

Where $\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials at most in the second order, and is a first – order polynomial. In order to find a particular solution of Eq. (2) we use the separation of variables with the transformation

$$\Psi(s)=\varphi(s)\chi(s) \qquad (3)$$

It reduces Eq. (2) to an equation ofhypergeometric type

$$\sigma(s)\chi''(s)+z(s)\chi'(s)+\lambda\chi(s)=0 \qquad (4)$$

and $\varphi(s)$ is defined as a logarithmic derivative in the following form and its solution can be obtained from

$$\frac{\varphi'(s)}{\varphi(s)}=\frac{\pi(s)}{\sigma(s)} \qquad (5)$$

The other part of the wave formation $\chi(s)$ is the hypergeometric type function whose polynomial solutions are given by Rodriques relations**.**

$$\chi_n(s)=\frac{B_n}{\rho(s)}\frac{d^n}{ds^n}\left[\sigma^n(s)\rho(s)\right] \qquad (6)$$

whereB_n is a normalization constant, and the weight function $\rho(s)$ must satisfy the condition.

$$\frac{d}{ds}(\sigma\rho)=\tau(s)\rho(s) \qquad (7)$$

with

$$\tau(s)=\bar{\tau}(s)+2\pi(s) \qquad (8)$$

The function $\pi(s)$ and the parameter λ required for the NU method are defined as follows:

$$\pi(s)=\frac{\sigma'-\bar{\tau}}{2}\pm\sqrt{\left(\frac{\sigma'-\bar{\tau}}{2}\right)^2-\bar{\sigma}+k\sigma} \qquad (9)$$

$$\lambda=k+\pi'(s) \qquad (10)$$

On the other hand in order to find the value of k, the expression under the square root of Eq. (9) must be square of polynomial. Thus, a new eigenvalue for the second order equation becomes

$$\lambda=\lambda_n=-n\frac{d\tau}{ds}-\frac{n(n-1)\sigma''}{2} \qquad (11)$$

Where the derivative $\frac{d\tau(s)}{ds}$ is negative.By comparison ofEqs. (10) and (11), we obtain the energy eigenvalues.

3.0 BOUND STATE SOLUTIONS OF THE SCHRODINGER EQUATION (SE) WITH INVERSE IQY POTENTIAL

For bound states with $l \neq o$,the radial SE equation reads

$$\frac{d^2\Psi}{dr^2}+\frac{2\mu}{\hbar^2}\left[E-V(r)-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]\Psi(r)=0 \qquad (12)$$

Where we have assumed that $R(r)=\frac{\Psi}{r}$ for the radial part of the SE,μ is the reduced mass,l is the angular momentum quantum number and V(r) is the IQY potential given in equation (1).

Using the transformation,

$$s=e^{-2br} \qquad (13)$$

Equation(12) with the potential can be written as:

$$\frac{d^2\Psi}{dr^2}+\frac{1}{s}\frac{d\Psi}{ds}+\frac{2\mu}{4(b\hbar)^2s^2}\left[E-\frac{Zs}{r^2}-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]\Psi(r)=0 \quad (14)$$

Equation (14) cannot be solved completely because of the centrifugal term $\frac{1}{r^2}$ for $l\neq 0$.In order to solve equation (14), we invoke an approximate scheme for centrifugal term [17] as

$$\frac{1}{r^2}=\frac{4b^2s}{(1-s)^2} \qquad (15)$$

Where we have used equation (13) in obtaining equation (15).With this approximation of the centrifugal term, equation (14) becomes,

$$\frac{d^2\Psi}{ds^2}+\frac{(1-s)}{s(1-s)}\frac{d\Psi}{ds}+\frac{1}{s^2(1-s)^2}[(-\varepsilon^2-\beta^2)s^2+(2\varepsilon^2-\gamma^2)s-\varepsilon^2]\Psi(s)=0 \quad (16)$$

Where we have employed the following dimensionless quantities:

$$\varepsilon^2=\frac{-\mu E}{2b^2\hbar^2};\beta^2=\frac{2\mu Z}{\hbar^2b^2};\gamma^2=l(l+1) \qquad (16)$$

Comparison of equations (16) and (2) reveals the following polynomials,

$$\bar{\tau}(s)=1-s,\sigma(s)=s(1-s);\bar{\sigma}(s)=(-\varepsilon^2-\beta^2)s^2+(2\varepsilon^2-\gamma^2)s-\varepsilon^2 \qquad (17)$$

Substituting these polynomials into equation (9), we obtain the four possible values as

$$\pi(s) = \frac{-s}{2} \pm \frac{1}{2}\begin{cases} \varepsilon s + \sqrt{(4\gamma^2 + 4\beta^2 + 1)} \\ \varepsilon s - \sqrt{(4\gamma^2 + 4\beta^2 + 1} \end{cases} \text{for} k_{\pm} = -\gamma^2 + \varepsilon\delta \quad (18)$$

Where $= \sqrt{(4\gamma^2 + 4\beta^2 + 1)}$.

For the polynomial, $\tau = \bar{\tau} + 2\pi(s)$,which has a negative derivative, we get

$$k_{-} = -\gamma^2 - \varepsilon\delta,$$

$$\pi(s) = \frac{-s}{2} - \frac{1}{2}\{\varepsilon s - \delta. \quad (19)$$

Hence we choose the proper value so that

$$\tau(s) = 1 - 2s - \varepsilon s + 2\delta, \quad (20)$$

Which has a negative derivative, with this selection, and $\lambda = k + \pi'$, we have

$$\lambda = -\gamma^2 - \varepsilon\delta - \frac{1}{2} - \frac{\varepsilon}{2}. \quad (21)$$

Now using Eq.(11),we have,

$$\lambda = \lambda_n = 2n + n(n-1) + n\varepsilon \quad (22)$$

Comparing Eq.s(21) and (22) we the energy spectrum for the Schrodinger equation with the IQY potential as

$$E_{n,l} = -\frac{2b^2\hbar^2}{\mu}\left[\frac{N + \frac{1}{2} + \gamma^2}{n + \frac{1}{2} + \delta}\right]^2 \quad (23)$$

Where N=2n+n(n-1).

Let us now find the wave function of the Schrodinger equation. Using the $\sigma(s)$and $\pi(s)$,we find the weight function as

$$\rho(s) = e^{-\nu(1-s)}(1-s)^{\nu-\mu-1} \quad (24)$$

and using Eq.(5),we obtain the wave function as

$$\varphi(s) = s^{\mu-1}(1-s)^{2\nu-\mu-1} \quad (25)$$

Where $\mu = 1 + \delta$ and $\nu = 2 + \varepsilon$.

We find the other wave function from Eq.(6) as

$$\chi_n = B_n e^{\nu(1-s)}(1-s)^{\mu-\nu+1}\frac{d^n}{ds^n}\left[s^n e^{-\nu(1-s)}(1-s)^{n+\nu-\mu-1}\right] \quad (26)$$

Hence, the wavefunction has the following form:

$$R_n(s) = N_n s^{\mu-1}(1-s)^{2\mu-\nu-1}p_n^{\nu,\mu-1}(1-2s) \quad (27)$$

Where N_nIs the new normalization constant determined by$\int_{-\infty}^{\infty} R_n^2(s)ds = 1$.

4.0 RESULT AND DISCUSSION

Two known potentials can be deduced from the IQY potential depending on the values of the screening parameter b. If $b \gg 1$, the IQY potential reduces to coulomb potential [13-22] given as

$$V(r) = -\frac{2Zb}{r} \quad (28)$$

And its energy eigenvalues and eigenfunctions have been obtain by NU method [14].

Similarly, if $b \ll 1$,the IQY reduces to the non-central potential [18],

$$V(r) = \frac{Z}{r^2} \quad (29)$$

This potential has been studied extensively in the literature and different methods [18] have been used to obtain their energy eigenvalues and eigenfunctions.

In order to show the accuracy of our results. We plotted the variation of the IQY potential as function of r for b=1 in Fig.1.We also plotted the variation of the energy spectrum with the quantum number n at Z=1 for different $l = 0.1,2,3$ in Fig.2. The energy spectrum is also plotted as a function of the atomic number Z for a fixed l and different $n = 0,1,2$ *and* 3 in Fig.3. Finally, we computed the energy eigenvalues of the IQY potential as function of the screening parameter for different Z-values and n-values in Table1 and Table 2.

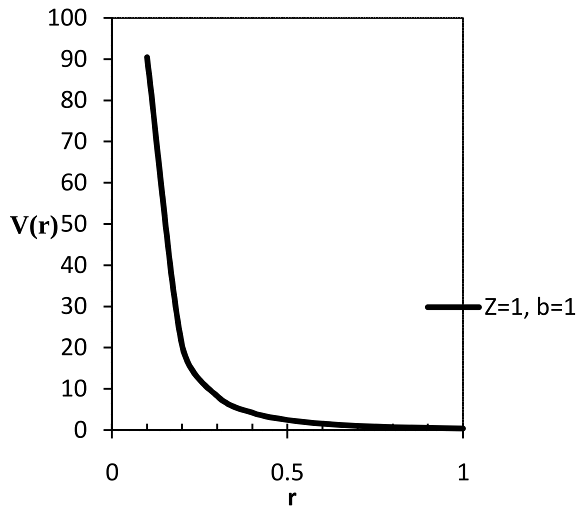
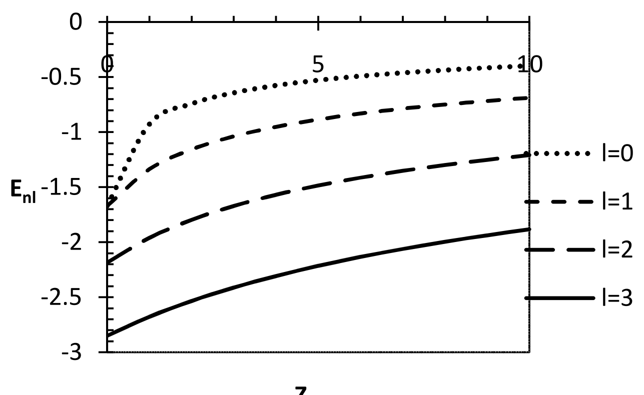


Fig. 1: A plot of IQY potential versus r for b=1, z=1



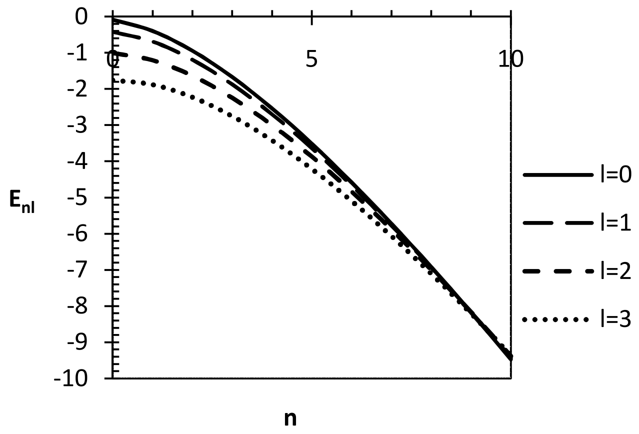


Fig. 3: A plot of energy spectrum E with n for Z=1 and l=0, 1, 2 and 3

Table 1: Energy eigenvalues of IQY potential as a function of the screening parameter for 2p, 4p, 4d and 4f,

State	b(fm)	Z	E
2p	0.914	40	-0.045
	1.026	50	-0.051
	1.153	60	-0.059
	1.242	70	-0.064
	1.322	80	-0.068
4p	0.721	10	-0.741
	1.242	15	-1.892
	1.321	20	-1.914
	1.434	25	-2.062
4d	1.032	35	-1.88
	1.144	40	-2.194
	1.232	45	-2.429
	1.324	50	-2.69
4f	1.293	70	-3.724
	1.325	80	-3.711
	1.39	90	-3.896
	1.421	100	-3.902

5.0 CONCLUSION

We obtain the solutions of the radial Schrodinger equation with IQY potential for an arbitrary l states using Nikiforov-Uvarov method. We obtain the energy eigen values and corresponding eigenfunctions of the IQY potential within the frame work of the NU method .We evaluated the energy eigenvalues as function of the screening parameter in table 1 and table2.Also,we display in figures1-3,the numerical results of our work. Finally, as it is presented our results are accurate for practical purposes.

6.0 ACKNOWLEDGEMENTS

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