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# Solutions of the Schrödinger Equation with Inversely Quadratic Yukawa Plus Mie-Type Potential using Nikiforov – Uvarov Method

**B. I. Ita**

Theoretical Quantum Mechanics Group, Department of Pure and Applied Chemistry, University of Calabar, Calabar, CRS, Nigeria

## ABSTRACT
The solutions of the Schrödinger equation with inversely quadratic Yukawa and Mie-type potential (IQYMP) for any angular momentum quantum number, $l$ have been presented using the Nikiforov-Uvarov method. The bound state energy eigenvalues and the corresponding un-normalized eigenfunctions are obtained in terms of the Laguerre polynomials. Special cases of the potential are also considered and their eigen values obtained.

## Keywords
Schrodinger equation, Inversely quadratic Yukawa potential, Mie-type potential, Nikiforov-Uvarov method.

## 1. INTRODUCTION
The bound state solutions of the Schrödinger equation (SE) are only possible for some potentials of physical interest [1-5]. Quite recently, several authors have tried to solve the problem of obtaining exact or approximate solutions of the Schrödinger equation for a number of special potentials [6-10]. Some of these potentials are known to play very important roles in many fields of Physics such as Molecular Physics, Solid State and Chemical Physics [8].

The purpose of the present work is to present the solution of the Schrodinger equation with the inversely quadratic Yukawa potential [11] plus Mie-type potential [12] of the forms

$V(r) = -V_o \frac{e^{-2\delta r}}{r^2}$ and $V(r) = -\frac{A}{r} + \frac{B}{r^2} + C$ respectively.

The sum of these potentials can be written as

$$V(r) = \frac{1}{r^2}(B - V_0) + \frac{1}{r}(2V_0\delta - A) + (C - 2V_0\delta^2) \quad (1)$$

Where $r$ represents the internuclear distance, $A, B$ and $C$ are constants, $\delta$ is the screening parameter and $V_0$ is the dissociation energy. Equation (1) is then amenable to Nikiforov-Uvarov method. Sever etal [13] have solved the Schrödinger equation with Mie potential and obtained the energy eigen values and their corresponding wave functions using Nikiforov-Uvarov method. Also, Ikhdair and Sever [14] found exact polynomial solution of the Mie-type potential in the N-dimensional Schrodinger equation. Arda and Sever [15] solved the Schrödinger equation with the Mie-type potential and pseudoharmonic potential via Laplace transform method and obtained energy eigen values and the wave functions. Few works appear in the literature on inversely quadratic potential [16 – 17]. However, not much has been achieved in the area of solving the radial Schrodinger equation for any angular momentum quantum number, $l$, with IQYMP potential using Nikiforov – Uvarov method in the literature.

## 2. NIKIFOROV-UVAROV METHOD
The Nikiforov-Uvarov (NU) method is based on the solutions of a generalized second-order linear differential equation with special orthogonal functions [18]. The Schrodinger equation of the type as:

$$\psi(z) + [E - V(z)]\psi(z) = 0 \quad (2)$$

could be solved by this method. This can be done by transforming equation (2) into an equation of hypergeometric type with appropriate coordinate transformation $z = z(r)$ to get

$$\psi(z) + \frac{\tilde{\tau}(z)}{\sigma(z)}\psi(z) + \frac{\tilde{\sigma}(z)}{\sigma^2(z)}\psi(z) = 0 \quad (3)$$

To find the exact solution to equation (3), we write $\psi(z)$ as

$$\psi(z) = \phi(z)\chi(z) \quad (4)$$

Substitution of equation (4) into equation (3) yields equation (5) of hypergeometric type as

$$\sigma(z)\chi(z) + \tau(z)\chi(z) + \lambda\chi(z) = 0 \quad (5)$$

In equation (4), the wave function $\phi(z)$ is defined as the logarithmic derivative [18]

$$\frac{\phi(z)}{\phi(z)} = \frac{\pi(z)}{\sigma(z)} \quad (6)$$

with $\pi(z)$ being at most first order polynomials. Also, the hypergeometric-type functions in equation (5) for a fixed integer $n$ is given by the Rodrigue relation as

$$\chi_n(z) = \frac{B_n}{\rho_n}\frac{d^n}{dz^n}[\sigma^n(z)\rho(z)] \tag{7}$$

where $B_n$ is the normalization constant and the weight function $\rho(z)$ must satisfy the condition

$$\frac{d}{ds}[\sigma^n(z)\rho(z)] = \tau(z)\rho(z) \tag{8}$$

With

$$\tau(z) = \bar{\tau}(z) + 2\pi(z) \tag{9}$$

In order to accomplish the condition imposed on the weight function $\rho(z)$ it is necessary that the polynomial $\tau(z)$ be equal to zero at some point of an interval $(a,b)$ and its derivative at this interval at $\sigma(z) > 0$ will be negative [18]. That is

$$\frac{d\tau(z)}{dz} < 0 \tag{10}$$

The function $\pi(z)$ and the parameter $\lambda$ required for the NU method are then defined as [20]

$$\pi(z) = \frac{\sigma - \bar{\tau}}{2} \pm \sqrt{\left(\frac{\sigma-\bar{\tau}}{2}\right)^2 - \bar{\sigma} + k\sigma} \tag{11}$$

$$\lambda = k + \pi(z) \tag{12}$$

The $z$-values in equation (11) are possible to evaluate if the expression under the square-root be square of polynomials. This is possible if and only if its discriminant is zero. Therefore, the new eigenvalue equation becomes [19]

$$\lambda = \lambda_n = -n\frac{d\tau}{dz} - \frac{n(n-1)}{2}\frac{d^2\sigma}{dz^2}, n = 0,1,2,\ldots \tag{13}$$

A comparison between equations (12) and (13) yields the energy eigen values.

## 3. SOLUTIONS OF THE RADIAL EQUATION

The radial Schrodinger equation is given as [19]

$$\frac{d^2R_{nl}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\lambda\hbar^2}{2\mu r^2}\right]R_{nl}(r) = 0 \tag{14}$$

Where $\lambda = l(l+1)$.

Equation (14) together with the potential in equation (1) and with the transformation $z = r^2$ yields the following equation:

$$\frac{d^2R(z)}{dz^2} + \frac{1}{2z}\frac{dR(z)}{dz} + \frac{1}{4z^2}(-\alpha z^2 + \beta z - \gamma)R(z) \quad = 0 \tag{15}$$

Where the radial wave function is $R(z)$ and

$$\alpha = -\frac{2\mu(E - C + 2V_o\delta^2)}{\hbar^2}, \beta = -\frac{2\mu(2V_o\delta - A)}{\hbar^2}, \gamma = \frac{2\mu(B - V_o)}{\hbar^2} + l(l+1) \tag{16}$$

Equation (15) is then compared with equation (3) and the following expressions are obtained

$$\tilde{\tau} = 2, \sigma(z) = z, \tilde{\sigma} = -\alpha z^2 + \beta z - \gamma \tag{17}$$

We then obtain the function $\pi$ by substituting equation (17) into equation (11):

$$\pi = \frac{1}{2} \pm \frac{1}{2}\sqrt{4\alpha^2 + (k - \beta)z + 1 + 4\gamma} \tag{18}$$

The discriminant of equation (18) gives

$$k_{\pm} = \beta \pm \sqrt{\alpha(1 + 4\gamma)} \tag{19}$$

When the two values of $k$ given in equation (19) are substituted into equation (18), the four possible forms of $\pi(z)$ are obtained as

$$\pi(z) = -\frac{1}{2} \pm \frac{1}{2}\begin{cases} 2\sqrt{\alpha}z + \sqrt{1+4\gamma}\ for \\ k_+ = \beta + \sqrt{\alpha(1+4\gamma)} \\ 2\sqrt{\alpha}z - \sqrt{1+4\gamma}\ for \\ k_- = \beta - \sqrt{\alpha(1+4\gamma)} \end{cases} \tag{20}$$

. Therefore, the most suitable expression of $\pi(z)$ is chosen as

$$\pi(z) = -\frac{1}{2} - \frac{1}{2}\left(2\sqrt{\alpha}z - \sqrt{1+4\gamma}\right) \tag{21}$$

From equation (13), $\lambda = \lambda_n$, we obtain the energy of the IQYMP as

$$E = C + 2V_o\delta^2 - \frac{\mu(2V_o\delta - A)^2/2\hbar^2}{\left(n + \frac{1}{2} + \sqrt{\frac{2\mu(B)}{\hbar^2} + \left(l + \frac{1}{2}\right)^2}\right)^2} \tag{22}$$

. The wave function $R(z)$ can be obtained in terms of the generalized Laguerre polynomials as

$$R(z) = N_n z^{(-1+\sqrt{1+4\gamma})/2} e^{-\sqrt{\alpha}z} L_n^{\sqrt{1+4\gamma}}(2\sqrt{\alpha}z) \tag{23}$$

$N_n$ is the normalization constant.

## 4. DISCUSSION

We now discuss special cases of the potential.

Case 1: If we set the parameters, $B = C = V_o = 0, A = ze^2$, it is easy to show that equation (22) reduces to the bound state energy spectrum of a particle in the Coulomb potential, i.e., $E_{np} = -Z^2\mu e^4/2\hbar^2 n_p^2$, where $n_p = n + l + 1$, is the principal quantum number.

Case 2: Similarly, if we set $A, B, C \neq 0, V_o = 0$, equation (22) results in the bound state energy spectrum of a vibrating-rotating diatomic molecule subject to the Mie-type potential as follows:

$$E = C - \frac{\mu(A)^2/2\hbar^2}{\left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2}\right)^2 + \frac{2\mu(B)}{\hbar^2}}\right)^2} \tag{24}$$

Case 3: If $C = V_0 = 0, A \neq 0, B \neq 0$ in equqtion (22) we obtain the energy of the Kratzer-Feus potential as

$$E = -\frac{\mu(A)^2/2\hbar^2}{\left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2}\right)^2 + \frac{2\mu(B)}{\hbar^2}}\right)^2} \tag{25}$$

Equations (24) and (25) are similar to the ones obtained in reference [12].

## 5. CONCLUSION

The Schrodinger equation with inversely Yukawa plus Mie-type potential has been successfully solved using the Nikiforov-Uvarov method. We have also obtained the energy eigen values and their corresponding un-normalized eigen functions. Special cases of the potential have been considered and their energy eigen values obtained.

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