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mbc-Optimal Codes

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ABSTRACT

In this paper, we present mixed burst correcting (*mbc*) optimal codes, that can correct all the bursts of length b_1 (fixed) in the first sub-block of length n_1 , all the bursts of length b_2 (fixed) in the second sub-block of length n_2 and all the bursts of length b_3 (fixed) in the last sub-block of length n_3 , along with the overlapping burst of length b (fixed) $(b \geq b_1+b_2, b \geq b_1+b_3)$ in two consecutive sub-blocks.

Keywords

mbc-codes, burst(fixed), error pattern-syndromes table, parity check matrix.

1. INTRODUCTION

Burst is the most common error in many communication systems. In sub-block oriented codes, burst can occur only with in the same sub-block. Correcting such burst error usually result into new type of linear codes or the most interesting perfect codes.

Most of the research activities have been followed for random error correction. Tietavainen and Perko [1] finally settled this problem by showing that there is no perfect codes other than the single error Hamming [7],double and triple error correcting Golay codes [5]. Many studies have been carried out for constructing blockwise burst error correcting (BBEC) perfect codes for the usual definition of burst according to which

"A burst of length b_1 in an (n,k) linear code is a vector whose all the non-zero components are confined to some b-consecutive positions, the first and last of which are non-zero".

There are many other systems where the error do occur but not near the end of the code words. This has bought the another definition of burst due to Chen and Tang [6] with a modification due to Dass [2], according to which

"A burst is a vector whose all the non-zero components are confined to some consecutive positions the first of which is non-zero"

According to this definition (1000000) is a burst of length 7 whereas (001000) will be a burst of length at most 4. This definition has been found very useful in error analysis experiments on telephone lines [4] and in channels where error normally do not occur near the end of a vector particularly when the burst of length is very large. Using this definition of burst, Dass and Tyagi [3] introduced BBEC codes, with the generalization given by Tyagi and Sethi, and construct $(n_{1b_1}, n_{2b_2}, n_{3b_3})$ linear as well as optimal codes [9], [10].

Definition. An $(n_{1b_1}, n_{2b_2}, n_{3b_3})$ code is a code that correct all the bursts of length b_1 (fixed) in the first sub-block of length n_1 , all the bursts of length b_2 (fixed) in the second sub-block of length n_2 and all the bursts of length b_3 (fixed) in the last sub-block of length n_3 .

Recently, Tyagi and Sethi [8] in an unpublished paper have studied lower and upper bound for codes that correct different bursts in different sub-blocks along with the overlapping bursts of a given length in two consecutive sub-blocks of a code word and them as named *mbc*-codes.

In this correspondence, we construct optimal *mbc*-codes in various cases based on the different sizes of sub-block and different length of the burst in both binary and non binary cases.

The necessary bound proved by Tyagi and Sethi [8] is as follows:

Theorem. The number of parity check digits for an *mbc*-code is atleast

$$q^{n-k} \geq 1+(q-1)[(n_1-b_1+1)q^{b_1-1}+(n_2-b_2+1)q^{b_2-1}+(n_3-b_3+1)q^{b_3-1}] + \frac{1}{2}(q-1)^2 q^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2) + \frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2) \qquad (1)$$

where $n_1+n_2+n_3$.

2. OPTIMAL CODES

The codes are optimal in the sense that these codes can be used to correct all the bursts of length b_1 (fixed) in the first sub-block of length n_1, all the bursts of length b_2 (fixed) in the second sub-block of length n_2 and all the bursts of length b_3 (fixed) in the last sub-block of length n_3, along with the overlapping burst of length b(fixed) $(b \geq b_1+b_2, b \geq b_1+b_3)$ in two consecutive sub-blocks and no more. Such codes are termed as ***mbc*-optimal codes.**

Considering the equality case of (1):

$$q^{n-k} = 1+(q-1)[(n_1-b_1+1)q^{b_1-1}+(n_2-b_2+1)q^{b_2-1}+(n_3-b_3+1)q^{b_3-1}] + \frac{1}{2}(q-1)^2 q^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2) + \frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2) \qquad (2)$$

For this equality (2), we have considered various cases for $q=2$ then considered for $q=3$ and $q=5$.

2.1. In case of $q=2$ i.e. in binary case: we have taken the main three cases of the various cases for the parameters n_1, n_2, n_3 and b_1, b_2, b_3

Case (i). When, $n_1=n_2=n_3=N$, $b_1=b_2=b_3=b'$ the given bound in (2) can be expressed as

$$2^{n-k} = 1+3(N-b'+1)2^{b'-1}+2^{2b'-2}(b-2b'+1)(b-2b'+2) \qquad (3)$$

Considering $b=3, N=3, b'=1,$ the equation (3) gives rise to (9, 5) optimal code that can correct all the single errors in all the sub-blocks and a burst of length 3 simultaneously in two consecutive sub-blocks. For this, consider the following parity check matrix.

$$H_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

It can be verified in the following table that the code is a *mbc*-optimal code as all the syndromes being distinct.

Table1

Error Pattern	Syndromes
100 000 000	0110
010 000 000	1000
001 000 000	0001
000 100 000	0010
000 010 000	0100
000 001 000	0111
000 000 100	1011
000 000 010	1001
000 000 001	1101
010 100 000	1010
001 100 000	0011
001 010 000	0101
000 010 100	1111
000 001 100	1100
000 001 010	1110

Case (ii). For $n_1 \neq n_2 \neq n_3$, $b_1=b_2=b_3=b'$, then equality (2) can be expressed as.

$$2^{n-k} = 1+2^{b'-1}(n-3b'+3)+2^{2(b'-1)}(b-2b'+1)(b-2b'+2) \qquad (4)$$

In this case, for $n = n_1 + n_2 + n_3 = 2 + 3 + 4 = 9$, $b' = 1$, $b = 3$, we have obtained a (9, 5) optimal code that may correct all single errors in all the three sub-blocks together with the burst of length 3(fix) simultaneously in the vector of length n_1+n_2 and n_2+n_3.

Consider the following parity check matrix

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Table 2

Error Pattern	Syndromes
10 000 0000	1000
01 000 0000	0001
00 100 0000	0010
00 010 0000	0100
00 001 0000	0111
00 000 1000	1011
00 000 0100	1001
00 000 0010	1101
00 000 0001	0110
10 100 0000	1010
01 100 0000	0011
01 010 0000	0101
00 010 1000	1111
00 001 1000	1100
00 001 0100	1110

Case (iii). If $b'=b_1=b_3 \neq b_2, n_1=n_2=n_3=N$, then equality (2) can be expressed as

$$2^{3N-k} = 1+2^{b_2-1}(N-b_2+1)+2^{b'}(N-b'+1)+2^{b_2+b'-2}(b-b_2-b'+1) \qquad (5)$$

for $N=3$, $b_2=1$, $b'=2$, $b=4$. The equation (5) gives rise to (9, 5) code. The parity check matrix for the code (9, 5) may be given as

$$H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

It can be verified from the error pattern-syndrome table (as above) that the syndromes of the errors are all distinct. So the code is a *mbc* optimal code.

2.2. In case of $q=3$ i.e. in ternary case, the equality (2) becomes

$$3^{n-k} = 1+2[(n_1-b_1+1)3^{b_1-1}+(n_2-b_2+1)3^{b_2-1}+(n_3-b_3+1)3^{b_3-1}]$$

$$+4\cdot 3^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2)$$
$$+4\cdot 3^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2) \qquad (6)$$

for $n_1=12$, $n_2=2$, $n_3=14$, $b_1=b_2=b_3=1$, $b=3$, $q=3$.
The equation (6) gives rise to (28, 24) code over GF(3). The parity check matrix of the (28, 24) code can be given as

$$H_4=\begin{bmatrix} 1&0&1&2&1&1&1&1&0&0&1&1&1&1&2&0&1&0&1&2&1&1&0&1&2&1&0&2 \\ 0&1&0&1&1&1&1&2&1&2&1&1&1&2&1&1&2&1&2&0&0&1&2&0&1&2&0&0 \\ 0&1&1&1&1&0&0&0&1&1&1&2&2&1&0&0&1&0&0&1&2&0&0&2&2&2&2&2 \\ 0&0&0&2&0&1&0&2&2&2&2&1&2&1&0&0&2&1&1&1&1&2&1&0&0&0&1&1 \end{bmatrix}$$

It can be verified from the Table 3 that the (28, 24) code is a mbc optimal code.

Table 3

Error Pattern		**Syndromes**	
100000000000	00	00000000000000	1000
200000000000	00	00000000000000	2000
010000000000	00	00000000000000	0110
020000000000	00	00000000000000	0220
001000000000	00	00000000000000	1010
002000000000	00	00000000000000	2020
000100000000	00	00000000000000	2112
000200000000	00	00000000000000	1221
000010000000	00	00000000000000	1110
000020000000	00	00000000000000	2220
000001000000	00	00000000000000	1101
000002000000	00	00000000000000	2202
000000100000	00	00000000000000	1100
000000200000	00	00000000000000	2200
000000010000	00	00000000000000	1202
000000020000	00	00000000000000	2101
000000001000	00	00000000000000	0112
000000002000	00	00000000000000	0221
000000000100	00	00000000000000	0212
000000000200	00	00000000000000	0121
000000000010	00	00000000000000	1112
000000000020	00	00000000000000	2221
000000000001	00	00000000000000	1121
000000000002	00	00000000000000	2212
000000000000	10	00000000000000	1122
000000000000	20	00000000000000	2211
000000000000	01	00000000000000	1211
000000000000	02	00000000000000	2122
000000000000	00	10000000000000	2100
000000000000	00	20000000000000	1200
000000000000	00	01000000000000	0100
000000000000	00	02000000000000	0200
000000000000	00	00100000000000	1212
000000000000	00	00200000000000	2121
000000000000	00	00010000000000	0101
000000000000	00	00020000000000	0202
000000000000	00	00001000000000	1201
000000000000	00	00002000000000	2102
000000000000	00	00000100000000	2011
000000000000	00	00000200000000	1022
000000000000	00	00000010000000	1021
000000000000	00	00000020000000	2012
000000000000	00	00000001000000	1102
000000000000	00	00000002000000	2201
000000000000	00	00000000100000	0201
000000000000	00	00000000200000	0102
000000000000	00	00000000010000	1020
000000000000	00	00000000020000	2010
000000000000	00	00000000001000	2120
000000000000	00	00000000002000	1210
000000000000	00	00000000000100	1220
000000000000	00	000000000000200	2110
000000000000	00	000000000000010	0021
000000000000	00	000000000000020	0012
000000000000	00	000000000000001	2021
000000000000	00	000000000000002	1021
000000000010	10	000000000000000	2201
000000000020	20	00000000000000	1102
000000000001	10	00000000000000	2210
000000000002	20	00000000000000	1120
000000000020	10	00000000000000	0010
000000000010	20	00000000000000	0020
000000000002	10	00000000000000	0001
000000000001	20	00000000000000	0002
000000000002	02	00000000000000	2002
000000000002	01	00000000000000	1001
000000000001	02	00000000000000	0120
000000000000	10	10000000000000	0222
000000000000	20	20000000000000	0111
000000000000	20	10000000000000	0211
000000000000	10	20000000000000	0122
000000000000	01	10000000000000	0011
000000000000	02	20000000000000	0022
000000000000	01	20000000000000	2111
000000000000	02	10000000000000	1222
000000000000	01	01000000000000	1011
000000000000	02	02000000000000	2022
000000000000	02	01000000000000	1111
000000000000	01	02000000000000	2222

2.3. In case of q=5 i.e. in 5-ary case, the equality (2) becomes

$$5^{n-k}=1+4[(n_1-b_1+1)5^{b_2-1}+(n_2-b_2+1)5^{b_2-1}+(n_3-b_3+1)5^{b_3-1}]$$
$$+8.5^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2)$$
$$+8.5^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2)$$
(7)

Example 2. For $n_1=n_2=11, n_3=1, b_1=b_2=b_3=1, b=2, q=5$, the equation (7) leads to exist of the code (23, 21) code over GF(5). Consider the following parity check matrix of the code

$$H_5=\begin{bmatrix} 1&2&3&4&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0 \\ 0&0&0&0&1&1&1&2&2&2&1&3&3&4&4&4&1&2&4&1&1&1&1 \\ 1&1&1&1&1&2&4&2&3&4&3&4&1&1&3&4&0&0&0&2&4&0&1 \end{bmatrix}$$

We can verify from the error syndromes table of this code as

above that the (23, 21) code is also a mbc optimal code.

3. OPEN PROBLEMS AND REMARKS

In this paper, we have shown the existence of *mbc*-optimal codes, with the help of examples, for binary and non binary case. However, the problem needs further investigation to find the possibilities of the existence of *mbc*-optimal codes for more than three sub-blocks .

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