IJTPC, Vol.14, June 2017.

JOURNAL OF THEORETICAL PHYSICS & CRYPTOGRAPHY

www.IJTPC.org/Publications

Modified Gravity with Cosmological constant Λ

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Abstract

In the present study, Bianchi type II universe in $f(R,T)$ theory of gravity with a term Λ has been investigated. We obtain the gravitational field equations in the metric formalism, which follow from the covariant divergence of the stress-energy tensor. The field equations correspond for a specific choice of $f(R,T)=f_1(R)+f_2(T)$, with the particular choice of the functions $f_1(R)=\lambda_1 R$ and $f_2(T)=\lambda_2 T$. Some physical and geometric properties of the model along with physical acceptability of the solutions have been also discussed in detail.

Keywords: Bianchi type II universe, variable Cosmological constant, $f(R,T)$ Gravity.

Introduction

The universe is expanding at an accelerating rate had been experimented proved by High redshift type Supernova Ia , cosmic microwave background (CMB) and clusters of galaxies [1-7]. The accelerated expansion of the universe is driven by the negative pressure of the dark energy. To explain the late-time accelerated expansion of the Universe, various modified gravities have recently been verified. Harko et al. [8] proposed $f(R,T)$ gravity theory by taking into account the gravitational Lagrangian as the function of Ricci scalar R and of the trace of energy-stress tensor T . They have obtained the equation of motion of test particle and the gravitational field equation in metric formalism both. The $f(R,T)$ gravity models can explain the late time cosmic accelerated expansion of the Universe. The dependence from the trace T may be induced by exotic imperfect fluids or quantum effects. Point like Lagrangian's for $f(R,T)$ gravity had been investigated by Myrzakulov [9] . Houndjo [10] considered the $f(R,T)$ gravity model that satisfies the local tests and transition of matter from dominated era to accelerated phase. Several authors [11-42] studied different cosmological models in $f(R,T)$ theory of gravity.

Motivated by the aforesaid discussion in this paper, Bianchi type II universe in $f(R,T)$ theory of gravity with a term Λ has been considered.

2. Formation of gravitational field equations of $f(R,T)$ gravity

The $f(R,T)$ theory of gravity is the generalization or modification of General Relativity (GR).
In this theory the modified gravity action is given by

$$s = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^4x + \int L_m \sqrt{-g}\, d^4x \quad (2.1)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar *R*, and of the trace *T* of the stress-energy tensor of the matter.
The stress-energy tensor of matter as defined by Landau et al. [43] is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{\mu\nu}}, \quad (2.2)$$

and its trace is given by $T = g^{\mu\nu}T_{\mu\nu}$.
Varying the action (2.1) with respect to the metric tensor components $g^{\mu\nu}$, the gravitational field equation of $f(R,T)$ gravity is obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + f_R(R,T)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \quad (2.3)$$

where $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ and $\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$. Here ∇_i is the covariant derivative and T_{ij} is usual matter energy-momentum tensor derived from the Lagrangian L_m.
The contraction of equation (2.3) yields

$$f_R(R,T)R + 3\Pi f_R(R,T) - 2f(R,T) = \left(8\pi - f_T(R,T)\right)T - f_T(R,T)\Theta$$

with $\Theta = g^{\mu\nu}\Theta_{\mu\nu}$. (2.4)

Combining equations (2.3) and (2.4) and eliminating the $\Pi f_R(R,T)$ term, we get

$$f_R(R,T)\left(R_{\mu\nu} - \frac{1}{3}Rg_{\mu\nu}\right) + \frac{1}{6}f(R,T)g_{\mu\nu} = \left(8\pi - f_T(R,T)\right)\left(T_{\mu\nu} - \frac{1}{3}Tg_{\mu\nu}\right) - f_T(R,T)\left(\Theta_{\mu\nu} - \frac{1}{3}\Theta g_{\mu\nu}\right) + \nabla_\mu\nabla_\nu f_R(R,T), \quad (2.5)$$

where $\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}$

It is mentioned here that these field equations depend on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. There are three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}. \quad (2.6)$$

Bearing in mind all above classes, Harko et. al. [8] derived the gravitational field equations that may be relevant in explaining some of the open problems of cosmology and astrophysics and they had also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of a function $f(T)$. Alvarenga et al. [44] studied the viability of $f(R,T)$ gravity according to the energy conditions. They presented the general formalism of $f(R,T)$ theory by putting out $f(R,T) = f_1(R) + f_2(T)$, where $f_1(R)$ and $f_2(T)$ be the function of curvature and the trace of the energy momentum tensor. Also, by substituting suitable constraint on the input parameter, they obtained $R + 2f(T)$ type model which satisfy the energy condition. Hence, in this paper it is focused on the second class, i.e.

$$f(R,T) = f_1(R) + f_2(T) \quad . \quad (2.7)$$

The gravitational field equation (2.3) becomes

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + \left(g_{\mu\nu}\Pi - \nabla_\mu\nabla_\nu\right)f_1'(R) = 8\pi T_{\mu\nu} + f_2'(R)T_{\mu\nu} + \left(f_2'(T)p + \frac{1}{2}f_2(T)\right)g_{\mu\nu}, \quad (2.8)$$

where the prime denotes the differentiation with respect to the argument.
Here the particular form of the functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$, where λ_1 and λ_2 are arbitrary constants is considered.
Here we take $\lambda_1 = \lambda_2 = \lambda$ so that $f(R,T) = \lambda(R+T)$. (2.9)
Above equation (2.8) can be written as

$$\lambda R_{\mu\nu} - \frac{1}{2}\lambda(R,T)g_{\mu\nu} + \left(g_{\mu\nu}\Pi - \nabla_\mu\nabla_\nu\right)\lambda = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \lambda\left(2T_{\mu\nu} + pg_{\mu\nu}\right), \quad (2.10)$$

Setting $\left(g_{\mu\nu}\Pi - \nabla_\mu\nabla_\nu\right)\lambda = 0$ we get

$$\lambda G_{\mu\nu} = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \left(\lambda p + \frac{1}{2}\lambda T\right)g_{\mu\nu}. \quad (2.11)$$

Equation (2.11) could be rearranged as

$$G_{\mu\nu} - \left(p + \frac{1}{2}T\right)g_{\mu\nu} = \left(\frac{8\pi + \lambda}{\lambda}\right)T_{\mu\nu}. \quad (2.12)$$

We have the Einstein field equation with cosmological constant

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu}. \quad (2.13)$$

Choosing a negative small value for the arbitrary λ so that we have the same sign of the RHS of equation (2.12) and (2.13), we keep this choice of λ throughout. The term $\left(p + \frac{1}{2}T\right)$ can now be regarded as a cosmological constant. So, in the framework of the $f(R,T)$ gravity, we can get the cosmological constant as a function of the equation of state parameter *w*, the energy density ρ and the trace of the stress-energy tensor. But since *w* and ρ are already included in *T* so we could just write

$$\Lambda = \Lambda(T) = p + \frac{1}{2}T. \quad (2.14)$$

The dependence of the cosmological constant Λ on the trace of the energy momentum tensor *T* has been proposed before by Poplawski [45] where the cosmological constant in the gravitational Lagrangian is a function of the trace of the energy-momentum tensor, considering the perfect fluid the trace of our model is $T = -3p + \rho$.

3. Metric and field equations

The spatially homogeneous but totally anisotropic and non-flat Bianchi type II space-time in the form

$$ds^2 = -dt^2 + R^2[d\theta^2 + f^2(\theta)d\phi^2] - S^2[d\psi + h(\theta)d\phi]^2, \quad (3.1)$$

where R & S are the functions of t only.

The energy-momentum tensor of the source is given by

$$T_i^j = (\rho + p)u_i u^j + p\delta_i^j , \quad (3.2)$$

where u^i is the flow vector satisfying

$$g_{ij}u^i u^j = -1 . \quad (3.3)$$

Here ρ is the total energy density while p is the corresponding pressure of a perfect fluid, which are related by an equation of state $p = \omega\rho$.

In a co-moving system of coordinates , from equation (3.2) we find

$$T_1^1 = T_2^2 = T_3^3 = p \text{ and } T_4^4 = -\rho . \quad (3.4)$$

From the equation of motion (2.13), the spatially homogeneous and totally anisotropic and non-flat Bianchi type-II universe (3.1) for the perfect fluid stress energy tensor (3.4) can be written as

$$\frac{\ddot R}{R} + \frac{\ddot S}{S} + \frac{\dot R}{R}\frac{\dot S}{S} + \frac{S^2}{4R^4} = \left(\frac{8\pi + \lambda}{\lambda}\right)(p) + \Lambda' \quad (3.5)$$

$$2\frac{\ddot R}{R} + \frac{\dot R^2}{R^2} - \frac{3S^2}{4R^4} = \left(\frac{8\pi + \lambda}{\lambda}\right)(p) + \Lambda \quad (3.6)$$

$$2\frac{\dot R}{R}\frac{\dot S}{S} + \frac{\dot R^2}{R^2} - \frac{S^2}{4R^4} = \left(\frac{8\pi + \lambda}{\lambda}\right)(-\rho) + \Lambda \quad (3.7)$$

where the overhead dot denotes differentiation with respect to *t*.

4. Solution of the field equations

The Einstein's field equations (3.5) to (3.7) are a coupled system of non-linear system of differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. There are only three independent equations with five unknowns R, S, p, ρ, Λ. In order to solve the field equation, the relation between the metric potentials is assumed which is given as follows

$$R = S^n , \quad (4.1)$$

where *n* is positive constant.

The physical significance of this condition for perfect fluid and barotropic EoS in a more general case has been discussed by Collins et. al. [46] . Many relativists [47-51] use above condition to find the exact solutions of cosmological models.

Using equations (4.1), (3.5) and (3.6) we obtain

$$2\ddot S + 2\frac{(n+1)}{S}\dot S^2 = \left(\frac{2}{1-n}\right)S^{2n-3} . \quad (4.2)$$

Let $\dot S = f(S)$ and $\ddot S = f\, f'$ where the prime denotes the differentiation with respect to *S*, so that above equation further reduces to

$$f^2 = \frac{1}{2n(1-n)}S^{2n-2} + c^2 S^{-2(n+1)} . \quad (4.3)$$

To find the solution of above equation, we consider $n = 2$, yields

$$f = \frac{1}{2S^3}\left(4c^2 - S^8\right)^{1/2} . \quad (4.4)$$

On integration of above equation (4.4), we obtain

$$R = \left(2c\sin(2t)\right)^{1/2} , \quad (4.5)$$

$$S^2 = \left(2c\sin(2t)\right)^{1/2} . \quad (4.6)$$

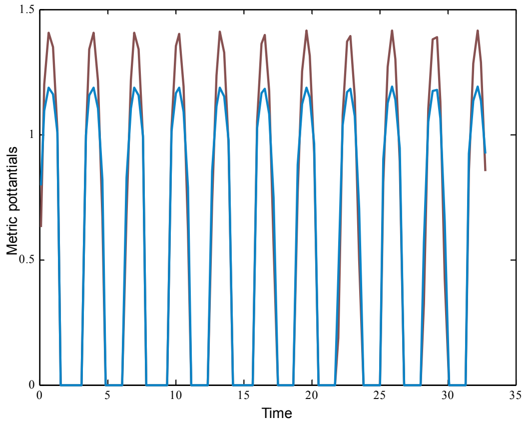

Figure 1: Behavior of metric potentials $(R \text{ and } S)$ verses time

.

5. Physical parameters of the universe

Energy density

$$\rho = \frac{\alpha_2(1-4\alpha_1)}{4(2c)^{3/2}\sin^{3/2}(2t)} - \frac{\alpha_2(4-\alpha_1)}{2}\cot^2(2t) - 2\alpha_1\alpha_2 \quad (4.7)$$

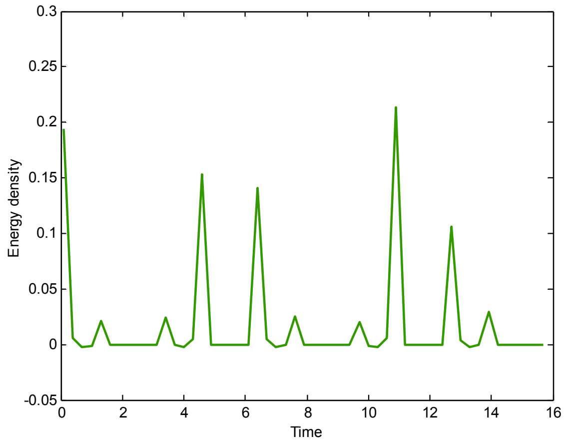

Figure 2: Energy density (ρ) vs time .

It is evident that the energy conditions $\rho \geq 0$ and $p \geq 0$ are satisfied under the appropriate choice of constants. From Eq. (4.8), it is observed that ρ is a positive decreasing function of time and it approaches to zero as $t \to \infty$ which resembles with the investigations of Katore et. al.[52]. This behavior is clearly depicted in Figure 2 as a representative case with appropriate choice of constants of integrations and other physical parameters using reasonably well known situations.

Anisotropic Pressure

$$p = 2\alpha_2(\alpha_1 - 2) - \frac{\alpha_2(\alpha_1 + 2)}{2}\cot^2(2t) - \left(\frac{\alpha_2(4\alpha_1 - 3)}{4}\right)\frac{1}{(2c)^{3/2}\sin^{3/2}(2t)}, \quad (4.8)$$

Cosmological constant

$$\Lambda = 2\alpha_1 + \frac{\alpha_1}{2}\cot^2(2t) + \frac{\alpha_1}{(2c)^{3/2}\sin^{3/2}(2t)}, \quad (4.9)$$

where $\alpha_1 = \dfrac{\lambda}{(8\pi)}$ and $\alpha_2 = \dfrac{\lambda}{(\lambda + 8\pi)}$.

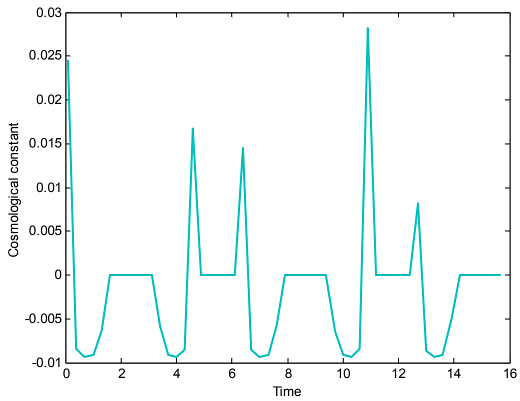

Figure 3: Cosmological constant (Λ) vs time .

The nature of Λ is clearly shown in figure 3 as a representative case with appropriate choice of constants of integration. It is observed that the cosmological term Λ is a decreasing function of time and it approaches a small positive value at late time. A positive value of Λ corresponds to a negative effective mass density (repulsion).Hence, it is expected that in the universe with a positive value of Λ the expansion will tend to accelerate whereas in the universe with negative value of Λ the expansion will slow down, stop and reverse. Recent cosmological observations suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-23}$. These observations on magnitude and red-shift of type-Ia supernova suggest that our Universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, our model is consistent with the results of recent observations.

6. Kinematical Properties of the Universe

The kinematical properties of the Universe that are important in cosmology are spatial volume V, expansion scalar θ, anisotropic parameter A_m, shear scalar σ^2 and Hubble parameter H, which have the following expressions

Spatial volume

$$V = 2c\sin(2t). \quad (4.10)$$

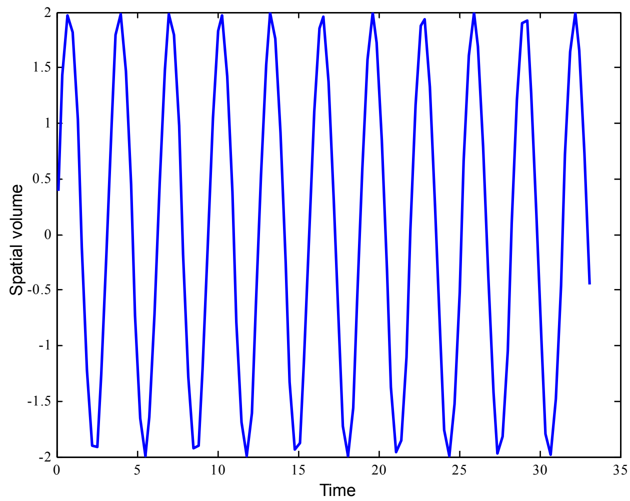

Figure 4: Spatial volume (V) vs time *t.*

At an initial epoch, the volume is zero which is shown in figure 4. The volume of the Universe is oscillatory and when time is zero we have big bang.

Hubble parameter

$$H = \frac{2}{3}\cot(2t) . \qquad (4.11)$$

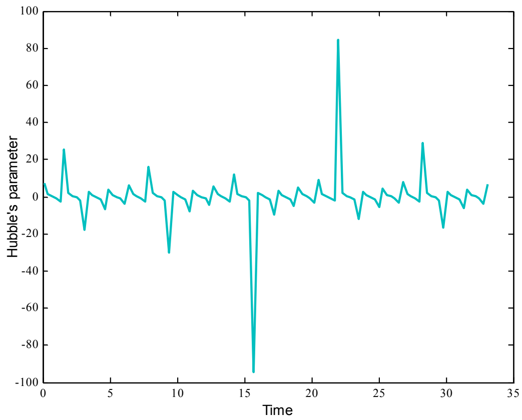

Figure 5: Hubble parameter (H) vs time.

Expansion Scalar

$$\theta = 2\cot(2t) . \qquad (4.12)$$

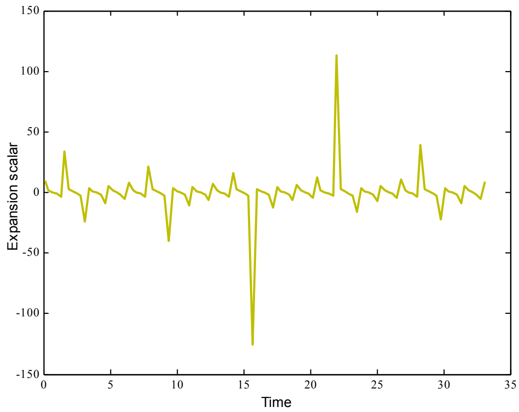

Figure 6: Expansion scalar (θ) vs time.

Shear Scalar $\sigma^2 = \frac{1}{12}\cot^2(2t)$. (4.13)

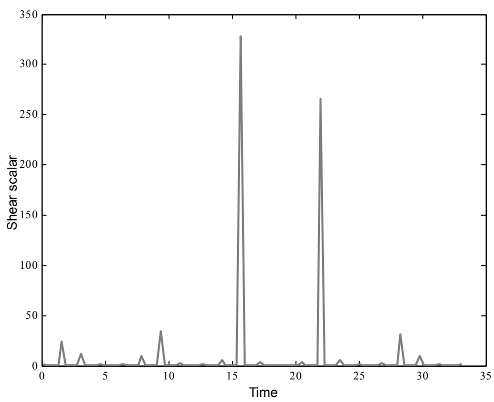

Figure 7: Shear scalar (σ^2) vs time.

The expansion scalar θ, the shear scalar σ and the Hubble parameter H decrease with the increase of time which is depicted in figures 5,6 and 7.

Anisotropic parameter $A_m = \frac{1}{8}$. (4.14)

The anisotropy is maintained throughout the evolution of the universe as the mean anisotropic parameter is constant.

Deceleration Parameter $q = \frac{-2}{3} + \tan^2(2t)$.

(4.15)

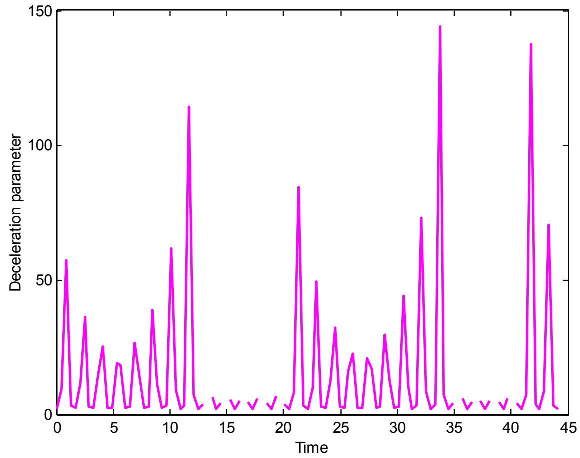

Figure 8: Deceleration Parameter vs time.

Whether the universe accelerates or decelerates is indicated by the sign of q . A positive sign of q corresponds to the standard decelerating model and the negative sign of q indicate acceleration. The universe is decelerating, as the value of the decelerating parameter is positive which is shown in figure 8.

7. Conclusions

Harko et al. [8] proposed a new theory known as $f(R,T)$ theory of gravity by modifying general theory of relativity to deal with the problems of late time acceleration of the universe . For this purpose, the class is taken as $f(R,T) = f_1(R) + f_2(T)$, with the individual superior functions $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$ and investigate the exact solutions of Bianchi type II cosmological model. It is observed that ρ is a positive decreasing function of time and it approaches to zero as $t \to \infty$. The volume of the universe is oscillatory and when time is zero we have big bang. Due to the positive sign of the deceleration parameter , the model obtained is decelerating initially and will accelerate in finite time due to cosmic recollapse (Nojiri and Odintsove [53]).The anisotropy is maintained throughout the evolution of the universe as the mean anisotropic parameter is constant. The nature of Λ in our derived model is supported by recent observations.

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